Nuclear Systematics: II. The Cradle of the Nuclides

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The nuclear energy surface is presented as three-dimensional (3-D) plots of mass or atomic number *versus* charge density *versus* average energy per nucleon. These plots reveal a trough or "cradle of the nuclides." Stable and long-lived nuclides are located in the valley. Those that are radioactive or are easily destroyed by fusion or fission occupy higher positions. Separate modes of nucleosynthesis fill the cradle from different directions and produce distinct occupancy levels, i.e., different abundance patterns. The cradle can be used to estimate the properties of nuclear matter that cannot be studied in the laboratory.

Introduction

This is the second paper in a series using nuclear systematics to try to decipher the origin, composition, and source of energy for the Sun. HARKINS[1] first proposed a relation between elemental abundance and nuclear structure in 1917, after finding that seven elements with even atomic numbers comprise 99% of the material in ordinary meteorites. The importance of atomic mass, in addition to atomic number, was stressed by RICHARDS[2] in 1919: *"If our inconceivably ancient Universe even had a beginning, the conditions determining that beginning must even now be engraved in the atomic weights. They are the hieroglyphics which tell in a language of their own the story of the birth or evolution of all matter, ..."* (p. 282).

In this paper we show that the ratio of atomic number to mass number, i.e., the nuclear charge density, is a useful way to display nuclides so the nuclear structure that determines elemental abundance can be visualized. When values of Z/A for the nuclides are presented on a three-dimensional (3-D) plot *versus* the mass or energy per nucleon, M/A, *versus* their mass number, A, neutron number, N, or atomic number, Z, the nuclides define the surface of a trough. The more stable nuclides lie along the bottom of the trough. Those that are radioactive or are easily destroyed by fusion or fission occupy higher positions in this trough that we call *the cradle of the nuclides*.

In the first paper[3] in this series charge density, Z/A, was one of the parameters considered to confirm the suggestion by HARKINS[1] that nuclear properties are related to elemental abundance, except for an anomalous spike of H in the Sun. That paper[3] also confirmed that Fe is the Sun's most abundant element[4], although Fe exists only as a trace element in the copious sea of H at the solar surface.

The most popular model for the origin of elements heavier than H is based on the pioneering work in 1957 by BURBIDGE et al.[5] (See also CAMERON[6]). They showed that fusion reactions may control stellar evolution and the synthesis of elements above H, including the production of an abundance peak at Fe in the terminal supernova (SN) explosion. BURBIDGE et al.[5] identified eight different types of nuclear reactions that might accompany stellar evolution and reproduce the 1956 elemental abundance estimate by SUESS and UREY[7].

Nuclear systematics

Figure 1 shows the ground states of all stable and radioactive nuclides from the latest edition of Nuclear Wallet Cards[8] on a three-dimensional (3-D) plot of mass number *versus* charge density *versus* mass (energy) per nucleon. When plotted in this manner[9], these nuclides reveal a *cradle of the nuclides* that is shaped like the trough made by cupping two hands together.

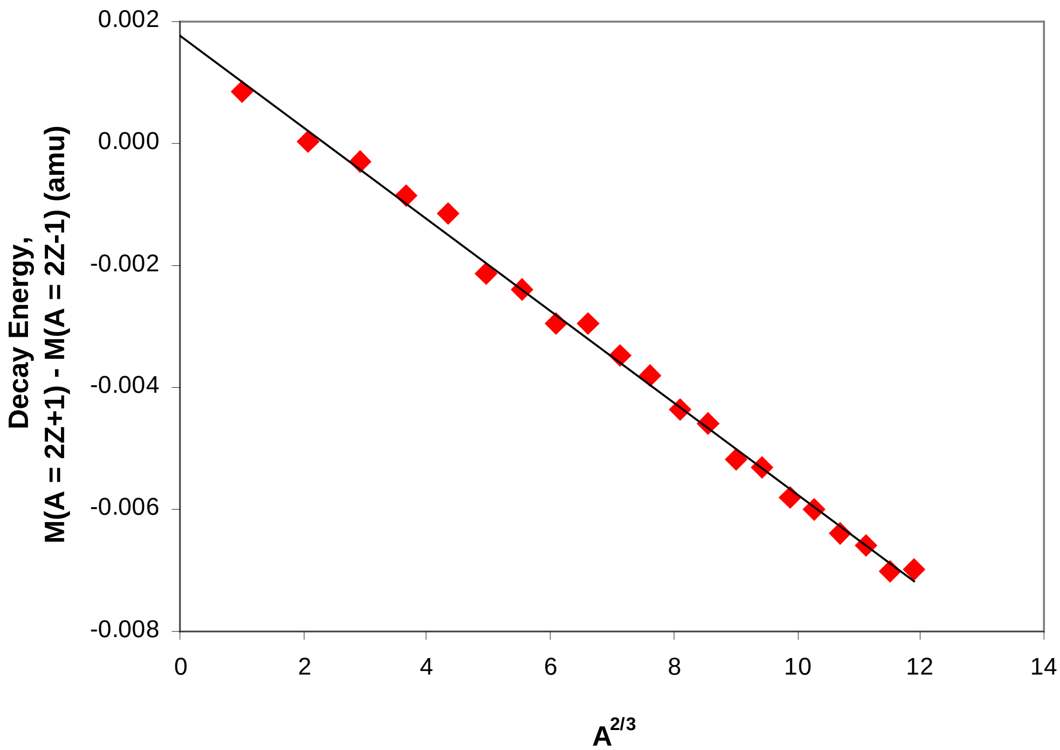


Figure 1

In Figure 1, stable nuclides occupy positions in the valley or lower region of the cradle. At any given value of A, stable isobars are near the bottom of the trough and radioactive isobars occupy higher positions on the sides. Neutron-rich isobars on the left side of the valley decay to lower positions by negatron emission; proton-rich isobars on the right side of the valley decay by electron-capture or positron emission.

Light-weight nuclides (low A) that are readily destroyed by fusion occupy relatively high positions in the cradle shown in Figure 1. The heavy nuclides (high A) that tend to decay by alpha-emission or by fission also populate elevated positions. Among stable nuclides, ^{1}H occupies the highest and ^{56}Fe the lowest, positions in the cradle. Thus, ^{1}H is the best fuel and ^{56}Fe the ash of ordinary thermonuclear fusion reactions.

Figure 2 shows a similar plot[9] of the ground states of all stable and radioactive nuclides on a 3-D plot of atomic number *versus* charge density *versus* mass (energy) per nucleon. At any given value of Z, the more stable isotopes *tend* to occupy the lower positions and the more radioactive isotopes *tend* to occupy higher positions, but the link between nuclear stability and mass per nucleon, M/A, is less certain for isotopes than for isobars.

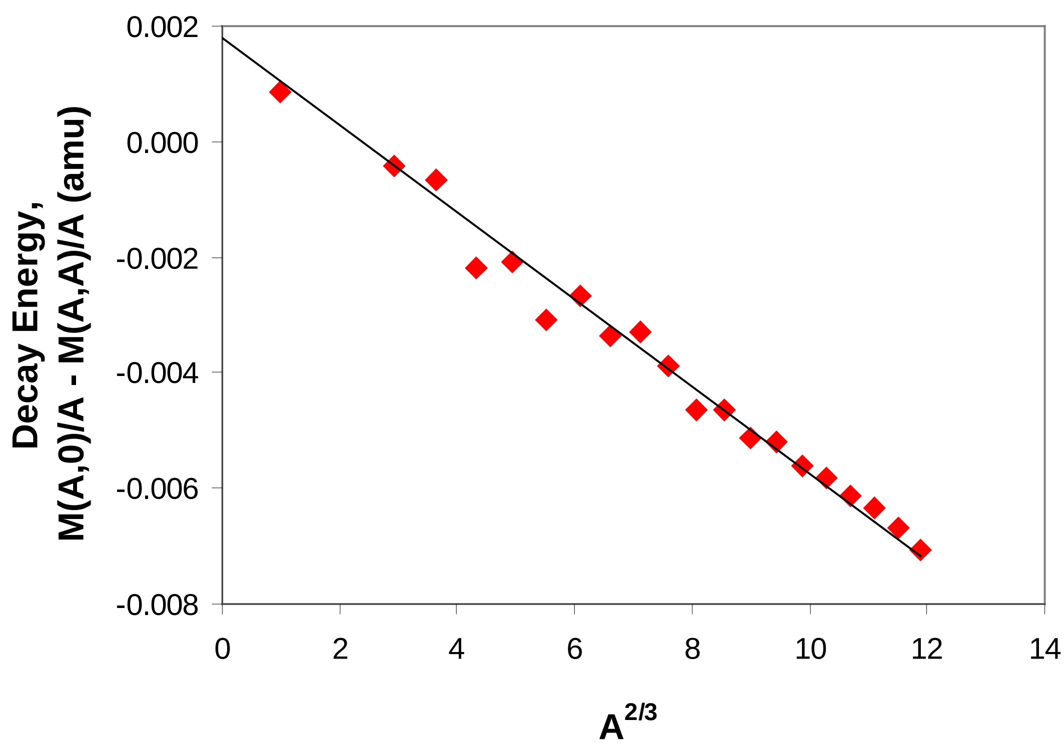


Figure 2

At any given value of Z, the number of nucleons decreases from left to right across the cradle. Extremely neutron-rich isotopes on the left side of the valley may decay by neutron-emission, but this is a rare process under normal conditions. Likewise, neutron-capture would transfer proton-rich isotopes on the right side into positions deeper in the valley.

Figure 3 shows the familiar mass parabola at A = 27 for an isobaric cut through the cradle displayed earlier in Figure 1. The neutron-rich isobars that decay by negatron-emission are on the left, and the proton-rich isobars that decay by electron-capture or positron-emission are on the right.

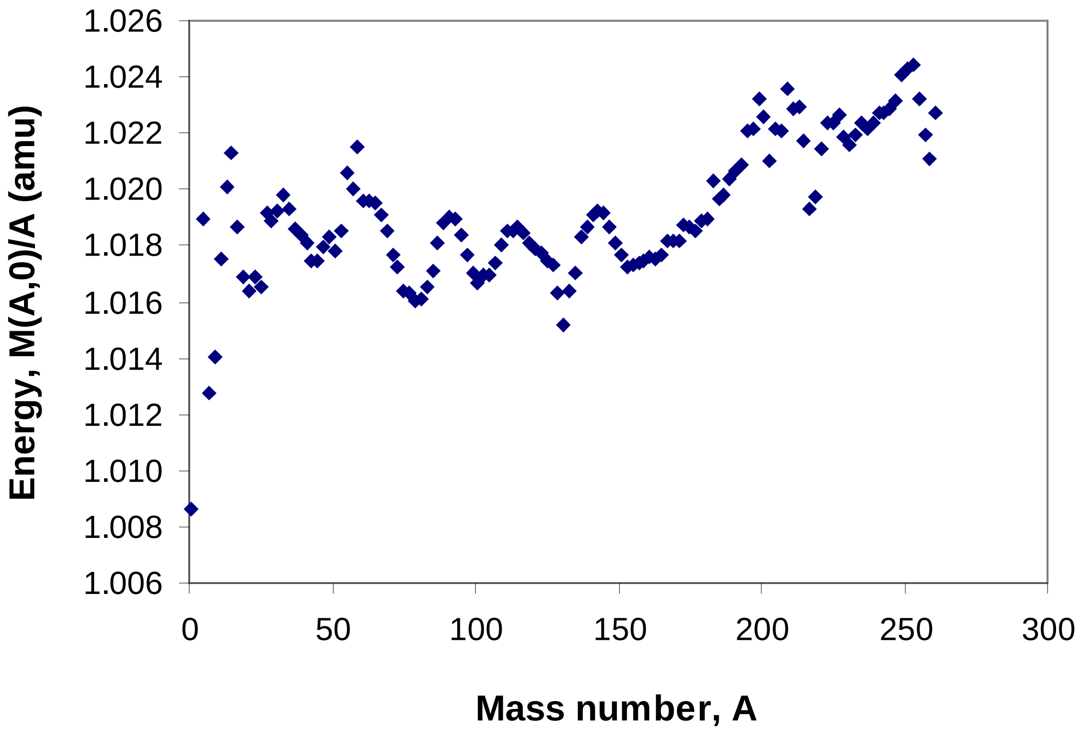


Figure 3

In Figure 3, the minimum value of M/A occurs at ^{27}Al. Extrapolation of the curve to the left, to the point where Z/A = 0, would correspond to a hypothetical nuclide composed of 27 neutrons. Likewise, extrapolation of the curve to the right, to the point where Z/A = 1.0, would correspond to a nuclide with 27 protons and 27 electrons, i.e., ^{27}Co.

Figure 4 shows the surface generated when isobaric curves, like that shown in Figure 3 at A = 27, are plotted through the data points at each value of A in Figure 1. This is a convenient way to show the general shape of the *cradle of the nuclides* without the clutter of individual data points.

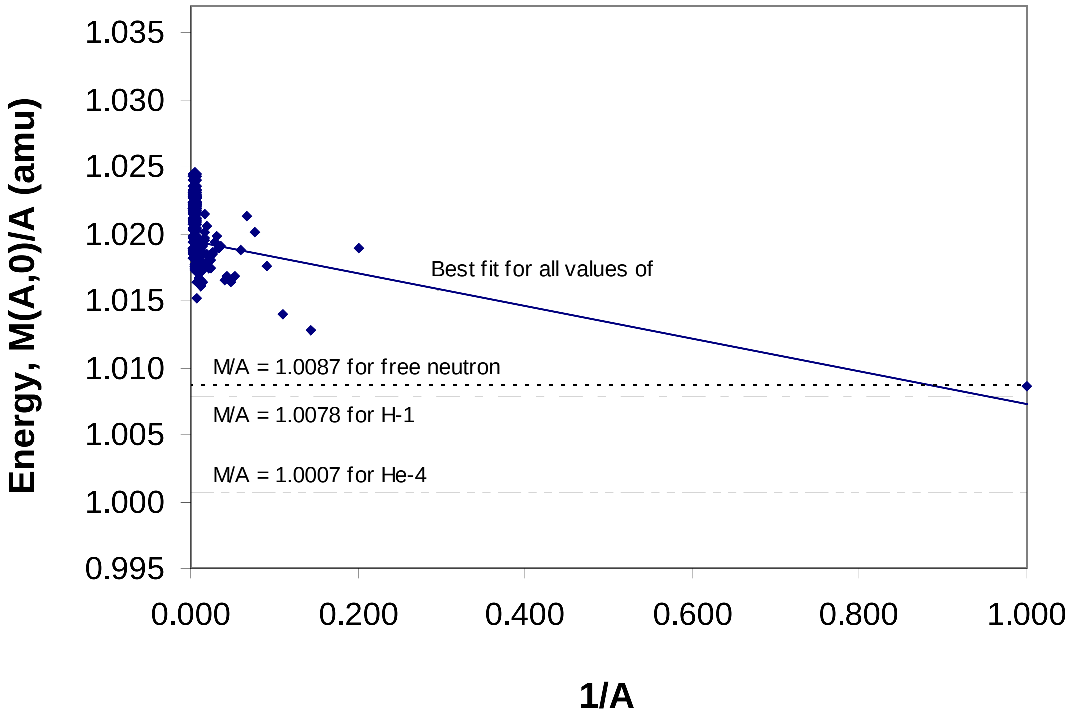


Figure 4

Figure 5 illustrates an isotopic cut, at Z = 27, through the cradle displayed earlier in Figure 2. Note that the data tend to lie along two parabolas. The lower one (filled symbols) is for even values of N; the upper one (open symbols) is for odd values of N. The number of nucleons decreases from left to right in Figure 5. Thus, the heavy, neutron-rich isotopes are on the left side of the trough and the light, neutron-poor isotopes are on the right side of the trough.

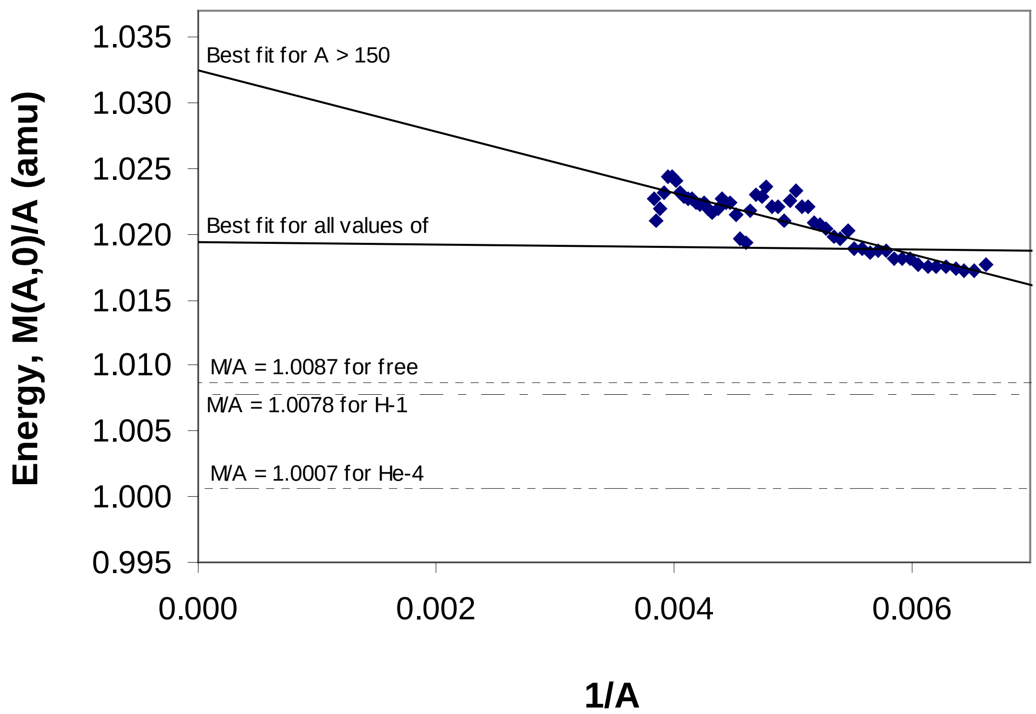


Figure 5

In Figure 5, the minimum value of M/A occurs at ^{59}Co. Extrapolation of the curves in Figure 5 to the right, to the point where Z/A = 1.0, would correspond to a nuclide with 27 protons and 27 electrons, ^{27}Co. Thus, the isobaric curve shown in Figure 3 and the isotopic curves shown in Figure 5 both lead to ^{27}Co. Likewise, extrapolation of the curves in Figure 5 to the left, to the point where Z/A = 0, would correspond to a massive nucleus composed entirely of neutrons except for 27 protons.

At Z = 27, ^{59}Co is the only stable isotope of cobalt, and this has the lowest value of M/A in Figure 5. However, some radioactive isotopes of other elements occupy lower positions (have lower values of M/A) than do specific stable isotopes. For example at Z = 54, the value of M/A for radioactive ^{122}Xe is higher than those of three stable isotopes - ^{124}Xe, ^{126}Xe, and ^{128}Xe - but lower than those of six other stable isotopes - ^{129}Xe, ^{130}Xe, ^{131}Xe, ^{132}Xe, ^{134}Xe and ^{136}Xe. Nevertheless, Figure 2 and the cross-section represented by Figure 5 illustrate the link between the charge density of isotopes and nuclear stability.

Figure 6 illustrates an isotonic cut through the data at N = 27. The data define two parabolas. The lower one (closed symbols) is for even values of Z; the upper one (open symbols) is for odd values of Z. The heavy, proton-rich isotones are on the right side of the trough in Figure 6 and the light, proton-poor isotones are on the left side of the trough.

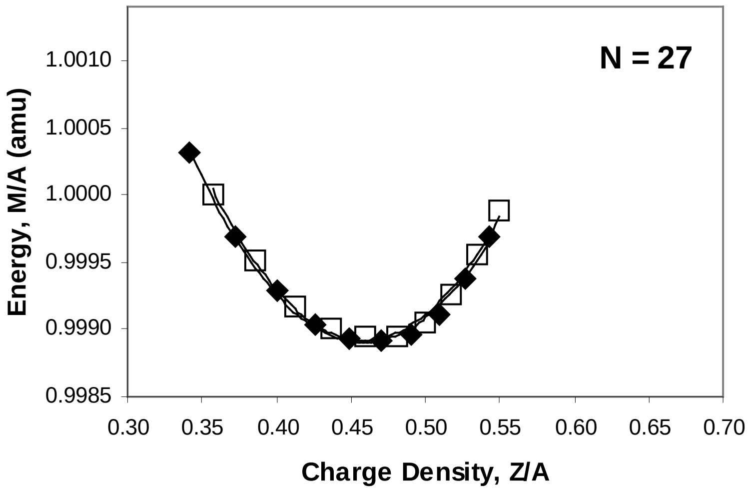


Figure 6

Extrapolation of the curves in Figure 6 to the left, to the point where Z/A = 0, would correspond to a nuclide containing only 27 neutrons. Thus, the mass per nucleon for a hypothetical nuclide composed of 27 neutrons can be obtained by extrapolating the isobaric curve shown in Figure 3, or the isotonic curves shown in Figure 6, to the intercept where Z/A = 0. Likewise, extrapolation of the curves to the right, to the point where Z/A = 1.0, would correspond to a massive nucleus composed entirely of protons except for 27 neutrons.

In Figure 6 the minimum value of M/A occurs in the region of Z/A = 0.45-0.47, corresponding to ^{49}Ti-^{50}V-^{51}Cr. There are actually two stable isotones at N = 27, ^{49}Ti and ^{50}V, but ^{51}Cr exhibits the lowest value of M/A. Thus, values of M/A alone do not predict relative stabilities of isotopes or isotones.

However, the 3-D cradles shown in Figures 1, 2 and 4 and the mass parabolas shown in Figures 3, 5 and 6 illustrate how well charge density sorts nuclides and/or subsets of isobars, isotopes and isotones in terms of average energy per nucleon (M/A). This is a major factor, but not the only one, in determining nuclear stability, and nuclear stability is not the only factor that determines abundance. Kinetics of the reactions that produced them also determine the abundance of nuclides, as will be discussed below.

Occupancy in the Cradle

The occupancy of different sites in the cradle of the nuclides provides a record of the processes that produced elements in the solar system. We will briefly review that record in terms of the classical papers on the synthesis of elements in stars[5,6].

BURBIDGE et al.[5] suggested that eight different types of nuclear reactions contributed to the synthesis of elements: H-burning, He-burning, α-process, p-process, e-process, s-process, r-process, and x-process. The first four of these are essentially fusion reactions that tend to produce light, proton-rich isotopes. The e-process is the violent, reactions that tend to populate the lowest positions in the cradle of the nuclides, producing an abundance peak centered on Fe. The next two reactions, the r- and s-processes, are neutron-capture reactions on rapid and slow time scales, respectively. Both produce neutron-rich nuclides. The r-process occurs rapidly in an excess of neutrons, making the neutron-rich isotopes of elements heavier than Fe and those elements heavier than Bi. The s-process operates close to the line of β-stability, producing many of the intermediate-mass isotopes of the ele-ments from Fe to Bi. The x-process, which was suggested as the source of D, Li, Be and B, was the least understood. Subsequent studies suggest that some of these, as well as ^{3}He and ^{4}He, were produced in the early universe[10]. We will not discuss here the occupancy of this region of the cradle.

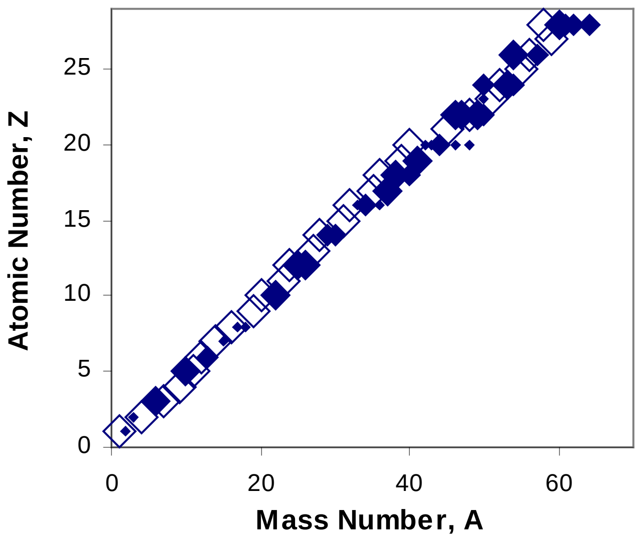
Figures 7a, 7b and 7c give a simple summary of the occupancy near the valley of the cradle of nuclides for stable elements in the solar system. These are displayed on plots of atomic number, Z, *versus* mass number, A. Figure 7a shows elements with Z = 1-28, 7b elements with Z = 29-56, and 7c elements with Z = 57-83.

The size of the symbols in these figures indicates relative abundance of the isotopes, which is better known than are the relative abundance of isobars, isotones or elements. The large open symbol designates the most abundant isotope at each value of Z. A correction has been made at Z = 18 for radiogenic ^{40}Ar from the decay of ^{40}K. No correction was made at Z = 82, since ^{208}Pb was the most abundant[11] primordial isotope of Pb.

The kinetic effects that determine abundance may be seen in the occupancy of nuclides in this plot of isotopes *versus* isobars. On all three graphs, the most abundant isotope is also frequently the only stable isobar. These nuclides could be produced by either fusion or neutron-capture reactions.

Coulomb barriers tend to limit fusion to light nuclides with low values of Z. In Figure 7a, above Z = 5 (i.e., by-passing those nuclides that were made in the early universe or by the e-process), the dominance of fusion reactions is suggested by high abundance's of the light, neutron-poor isotopes up to Sc, Z = 21. Above Sc, Z = 22-28, the distribution shifts presumably due to role of the e-process in producing nuclides in this region.

In Figures 7b and 7c, the neutron-rich isotopes become increasingly dominant for the heavier elements. Closed shell effects may explain some of the fine structure not discussed in this brief overview. For example, the high abundance of ^{16}O, ^{40}Ca, ^{56}Fe, ^{132}Xe, and ^{208}Pb may be related to the unusual stability of doubly magic nuc-lides made during nucleosynthesis, including precursor nuclides like ^{56}Ni and ^{132}Sn.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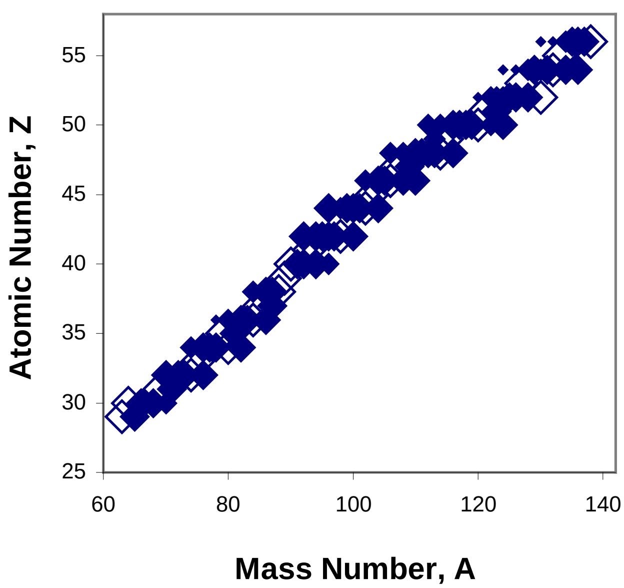
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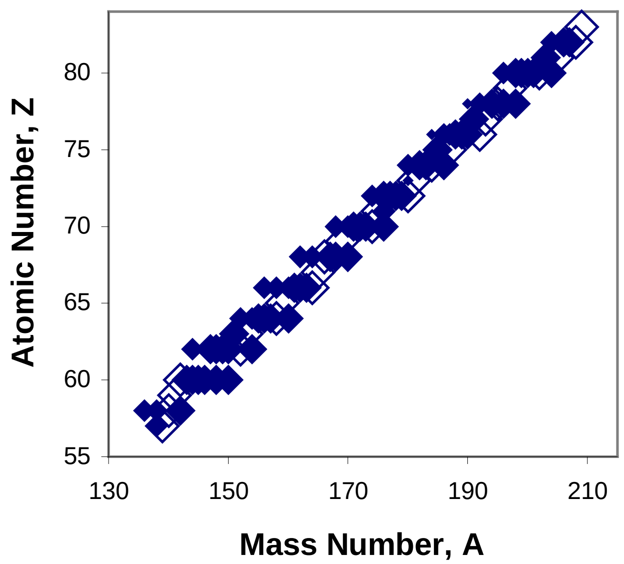

Figures 7a, 7b and 7c

Conclusions

In the first paper[3], it was concluded that the solar abundance of most elements is related to nuclear stability. The (3-D) plots of the nuclear energy surface (Figures 1, 2, and 4) and the cuts through that at constant A, Z and N (Figures 3, 5 and 6) offer a simple visualization of nuclear stability. Among stable nuclides, ^{1}H occupies the highest position and ^{56}Fe the lowest position in the *cradle of the nuclides*. In the next paper we will show how the cradle can also be used to estimate the properties of nuclear matter that might be produced under extreme conditions.

To explain the dominance of hydrogen in the 1956 abundance estimate by SUESS and UREY[7], BURBIDGE et al.[5] proposed that the SN debris produced by the terminal explosion of a massive star was mixed back into the interstellar medium before the solar system formed. The finding that meteorites trapped short-lived radioactivities and poorly-mixed isotopes of many elements suggests that these objects may actually have formed before the SN debris[12] was mixed with H-rich material of the interstellar medium.

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References

1. W. D. HARKINS, J. Am. Chem. Soc., 39 (1917) 856-879.

2. T. W. RICHARDS, "Atomic weights", Nobel lecture, December 6, 1919. Published in Nobel Lectures in Chemistry, 1901-1921, Elsevier Publ. Co., New York, NY, 1966, pp. 280-292.

3. O. MANUEL, C. BOLON, "Nuclear systematics: I. Solar abundance of the elements", J. Radioanal. Nucl. Chem., (2001) submitted for publication.

4. O. K. MANUEL, G. HWAUNG, Meteoritics, 18 (1983) 209-222.

5. E. M. BURBIDGE, G. R. BURBIDGE, W. A. FOWLER, F. HOYLE, Rev. Mod. Phys., 29 (1957) 547-650.

6. A. G. W. CAMERON, Publ. Astron. Soc. Pacific, 69 (1957) 201-222.

7. H. E. SUESS, H. C. UREY, Rev. Mod. Phys., 28 (1956) 53-74.

8. J. K. TULI, Nuclear Wallet Cards, Sixth Edition, National Nuclear Data Center, Brookhaven National Lab., Upton, NY, 2000, 74 pp.

9. O. MANUEL, C. BOLON, M. ZHONG, P. JANGAM, "The Sun's origin, composition and source of energy," Report to FCR, Inc., December 2000, http://www.umr.edu/~om/report_to_fcr/report_to_fcr1.html

10. G. WALLERSTEIN et al., Rev. Mod. Phys., 69 (1997) 995-1084.

11. C. PATTERSON, Geochim. Cosmochim. Acta, 10 (1956) 230-237.

12. O. K. MANUEL, J. T. LEE, D. E. RAGLAND, J. M. D. MACELROY, B. LI, W. K. BROWN, J. Radioanal. Nucl. Chem., 238 (1998) 213-225.

Figure captions

Figure 1. The ground states of all stable and radioactive nuclides[8] on a three-dimensional (3-D) plot[9] of mass number, A, *versus* charge density, Z/A, *versus* mass (energy) per nucleon, M/A.

Figure 2. The ground states of all stable and radioactive nuclides[8] on a 3-D plot[9] of atomic number, A, *versus* charge density, Z/A, *versus* mass (energy) per nucleon, M/A.

Figure 3. An isobaric cut, at A = 27, through the cradle of nuclides that is shown in Figure 1. The minimum value of M/A occurs at ^{27}Al for the A = 27 isobars.

Figure 4. The surface of the cradle of the nuclides made by passing isobaric mass parabolas at each value of A through the data shown in Figure 1. Estimates of nuclear masses for unknown nuclides at A = 2 and 3 were used to show approximate parabolas through the data points of ^{2}H, ^{3}H and ^{3}He.

Figure 5. An isotopic cut at Z = 27 through the cradle displayed in Figure 2. The minimum value of M/A occurs at ^{59}Co for the Z = 27 isotopes.

Figure 6. An isotonic cut through the cradle of the nuclides at N = 27. The minimum value of M/A occurs at ^{51}Cr for the N = 27 isotones.

Figure 7. The occupancy of Z and A sites along the valley of the cradle of the nuclides for stable isotopes in the solar system. The symbol size at each value of Z reflects isotopic abundance in that element. The most abundant isotope at each value of Z is shown by an open symbol.